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Commentary on Appendix L: Alignment of the Next Generation Science Standards with the Common Core State Standards for Mathematics

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Foreword

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The Next Generation Science Standards (NGSS) were released in April 2013. As we did for the two public drafts that preceded it, and for the NRC framework on which it is based, we asked our expert reviewers to evaluate these standards against our criteria for content, rigor, clarity, and specificity.

We released that evaluation in June (http://www.edexcellence.net/publications/final-evaluation-of-NGSS.html). It was the considered opinion of the review team, based on our criteria, that the NGSS deserve a "C" grade. Among the serious challenges the analysts found in the NGSS was an acute dearth of math content, even in situations where math is essential to the study and proper understanding of the science that students are being asked to master.

This failure to include essential math content is particularly troubling in light of statements by the authors of the NGSS that they intended to integrate mathematics into their new science expectations.

What's more, the authors assert that the NGSS

> are aligned, by grade level and cognitive demand, with the English Language Arts and Mathematics Common Core State Standards. This allows an opportunity both for science to be a part of a child's comprehensive education as well as ensuring an aligned sequence of learning in all content areas. The three sets of standards overlap and are reinforcing in meaningful and substantive ways.

But are they truly aligned? Did the authors successfully fit the math expectations of the NGSS with those of the Common Core—the latter having already been adopted by forty-five states and the District of Columba? And, if not, what does that mean for instruction in science—and in math—in the nation's hundreds of thousands of classrooms?

Reviewing math alignment

To determine how well NGSS math is aligned with the Common Core State Standards in Mathematics (CCSSM), we asked eminent Johns Hopkins mathematician and veteran Fordham reviewer W. Stephen Wilson to take a close look—and we are pleased now to present the results of his analysis.

Dr. Wilson focused on two areas: First, Wilson reviewed the "Connections to the Common Core" section that appears within the main NGSS document and lists pertinent Common Core standards. Second, Dr. Wilson reviewed *Appendix L: Connections to the Common Core State Mathematics Standards*, which was released by Achieve some weeks after the main document. That appendix was intended to provide concrete examples of how the math "connections" identified in the NGSS might play out usefully in science classrooms. It generally includes the full-text science expectation, the full text of the related math standard(s), and one or more "science examples" that demonstrate how grade-appropriate math content might enhance science learning.

Findings

Dr. Wilson found several important strengths—and a distressing number of weaknesses. On the positive side, where the NGSS document itself links to CCSSM expectations, Appendix L extends these connections with clear and explicit science examples that demonstrate how grade-appropriate mathematics can be used to enhance and elucidate the science. Appendix L also suggests math applications in places where the NGSS authors failed to do so. Indeed, it is clear that the authors of Appendix L struggled to backfill math content that *should* have been included in the NGSS proper but was not.

Unfortunately, the utility of Appendix L is seriously compromised by three types of shortcomings:

- In several cases where NGSS expectations require math in order to fully understand the science content, that math goes well beyond what students would have learned in classrooms aligned to the Common Core. In other words, the math in the NGSS and the math in the CCSSM are *not* fully aligned.
- Appendix L misses several opportunities to build important links between grade-appropriate math and required science content.
- Appendix L too often makes "superficial connections," in which grade-appropriate math is presented in ways that do little to enhance science learning.

Given the critical overlap between science and math, as well as the NGSS authors' intention to align their science expectations with the Common Core math standards, these shortcomings signal a need for caution on the part of states that are serious about implementing the CCSS but that are also considering adopting the NGSS.

Implications

As we've noted on several occasions, most states have weak science standards today—not to mention weak science instruction and weak science achievement—and would benefit greatly from a top-to-bottom overhaul of science education. But that is a large undertaking that should be embarked upon with great care.

If new academic standards—any new standards—are to gain traction in actual classrooms, their implementation must be taken very seriously. This involves curriculum, pedagogy, instructional materials, teacher preparation, assessment, and more. (In science, it may also involve facilities.) If a state—or district or school—is unwilling or unable to make these many changes, new standards won't lead to improved student learning. This point is particularly consequential for states that have already embarked upon the heavy lifting portended by the Common Core standards for math and English language arts. They should ask themselves whether they have the capacity to undertake simultaneous changes in other subjects.

If they are bent on replacing their science standards, the NGSS is not the only available option, nor is it necessarily the best one. We've noted previously six states received grades of A or A- from Fordham's reviewers for their present science standards. Other states might do well simply to adopt *their* standards. (We're particularly admiring of those of South Carolina.)

In considering NGSS as the alternative, state officials may wish to examine our review of their content, rigor, and clarity. But they would also be wise to read *this* review of NGSS math and its alignment (or lack thereof) with the CCSSM. They should understand that alignment between these two documents is flawed; that the NGSS is generally short on math, including math that's necessary if one is to truly understand and apply the science; that if they're going to replace their current standards with the NGSS, they should definitely embrace and apply Appendix L, too, not treat it as some sort of arcane backgrounder to put on the shelf; and that, even with the help of Appendix L, the NGSS feature missed opportunities and flawed applications of mathematics. They should be aware that classroom teachers may face challenges in trying simultaneously to deploy the CCSSM and the NGSS, because they don't quite line up.

Of course there are possible remedies. (The best would be a revised edition of the NGSS itself, but we don't see that happening anytime soon.) Treating Appendix L as an essential supplement to the NGSS will help. There could also be NGSS "supplements" developed—akin to "patches" for flawed computer-operating systems and software programs—that would resolve the NGSS-CCSSM differences, add math in crucial places to the NGSS, and turn the opportunities missed by Appendix L into opportunities seized. (In an ideal world, such supplements would also supply the important science content that was omitted in the NGSS.) We sincerely hope that someone will consider such supplementation. We also hope that states embarking on the NGSS will do so with their eyes wide open to the challenges and glitches that inevitably will follow.

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Summary

The authors of the Next Generation Science Standards (NGSS) made clear from the outset their intention to align the science standards with the Common Core State Standards in Mathematics (CCSSM). Since math curriculum and instruction in forty-five states and the District of Columbia will now be aligned to the Common Core, agreement between these two critical and closely related subjects is necessary to ensure that students aren't asked to do in science something they haven't yet learned in math—and that science education takes advantage of the content that students should have already learned in math.

Unfortunately, judging the alignment between the NGSS and the CCSSM is difficult for two reasons. First, as our science experts extensively documented in Fordham's evaluation of the NGSS, one of the salient shortcomings of the NGSS is their failure to include very much math at all, particularly in areas where mastery of related math content is essential to understanding science required by the NGSS. Second, while the main NGSS document does attempt to make connections with the CCSSM in a section called "Common Core State Standards Connections," those connections are superficial at best. Indeed, the "connections" amount to nothing more than a list of Common Core math standards that might be relevant to the science expectations. The authors provide no additional information about how educators might usefully integrate—or connect—the science and math.

In order to deliver more guidance on the alignment between the NGSS and the Common Core, a separate appendix (Appendix L, "Connections to the Common Core State Standards for Mathematics") was recently released. While the guidance provided in Appendix L is markedly more useful than the "connections" column in the main NGSS document, it too is insufficient on several counts.

First, because the NGSS themselves fail to integrate math properly into their science performance expectations, the authors of Appendix L were forced to retrofit the math that *should* have been in the standards into this ancillary document—which may or may not get used in conjunction with the standards document itself.

Second, the science-performance expectations within the NGSS are frequently so vague that they only hint at what students should know and be able to do. That means the authors of Appendix L also had to try to interpret each performance expectation to discern the science content that students will need to learn, then decide what mathematics would most appropriately support that science content for the given grade level. Take, for example, the following standards from grade 2, middle school, and high school:

- 2-PS1-2. Analyze data obtained from testing different materials to determine which materials have the properties that are best suited for an intended purpose.
- MS-LS3-1. Develop and use a model to describe why structural changes to genes (mutations) located on chromosomes may affect proteins and may result in

harmful, beneficial, or neutral effects to the structure and function of the organism.

- HS-ESS2-6. Develop a quantitative model to describe the cycling of carbon among the hydrosphere, atmosphere, geosphere, and biosphere.

These are so generic that any of them could be used to describe recent results from cutting edge research by senior scientists. That was presumably not the intent for K–12 science study, but that also means that readers and users of such standards must determine how to adapt and apply such cosmic expectations to students at the appropriate grade level. And, in the case of Appendix L, it's left to the authors both to interpret the standards and then to retrofit the math that would best support student learning at the appropriate grade.

This review focuses on the alignment of the NGSS with the CCSSM, which is made first in a "connections-to-the-CCSS" column in the NGSS. There, Common Core math standards that are related to specific science standards are merely listed, with no additional details about what mathematics could be used and how it could be taught with the science. Appendix L often takes this connection an important step further and offers excellent examples that allow mathematics to support and enhance the teaching and learning of science. Unfortunately, the appendix also suffers from three serious drawbacks. First, some science standards are quite explicit about the need for mathematics that (in the CCSSM) is beyond the grade level of the science standard. These situations are clear examples of a failure to align the NGSS with the CCSSM, and they are not problems that Appendix L could solve.

Second, Appendix L misses some opportunities to make meaningful connections between science and math.

Third, while the appendix is designed to illustrate connections between science and math, some of those connections are superficial. While grade appropriate, the math in those examples does little to support the science standards to which it's tied.

Organization of the Math Content in the NGSS

For grades K–5, science-performance expectations in the NGSS are offered for each grade level and can be organized either by "disciplinary core idea" (i.e., "Earth's Systems," "Energy," and "Matter and its Interactions") or by "topic" (i.e., "Forces and Interactions," "Weather and Climate," and "Structure and Function"). For grades 6–12, science-performance expectations are presented by grade band (Middle School or High School), and then they are grouped either by DCI or by topic. (Note that readers can choose to read the NGSS organized either by DCI or by topic. While the organization is different, the content presented is the same.)

In addition to the science content and skills, the NGSS present connections to the Common Core math (and English) content in two ways. First, beneath each topic or DCI, and for each grade level or grade band, the authors have listed "Common Core State Standards Connections" for English language arts and math. These connections include mathematical practices or standards drawn directly from the CCSSM, and they are meant to draw teachers' attention to relevant math that could be taught with the science content.

The NGSS proper are also now augmented by Appendix L, "Connections to the Common Core State Standards for Mathematics." Here, organized only by DCI, connections between the CCSSM and the NGSS are explained and, for many standards, an explicit "science example" is given. That example is intended to illustrate how the math could be used to deepen understanding of the science content or extend learning. Take, for example, the following Earth and Space Science standard (5-ESS2-1):

> "Develop a model using an example to describe ways the geosphere, biosphere, hydrosphere, and/or atmosphere interact."

In the main NGSS document, under "Common Core State Standards Connections," this standard is linked to a related fifth-grade math standard (CCSSM 5.G.A.2). In Appendix L, that "connected" math standard is quoted alongside a "science example." (Note that the bold-faced "science examples" appear only in Appendix L, not in the NGSS itself or in the CCSSM proper.)

> 5-ESS2-1 Earth's Systems
>
> 5.G.A.2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. **Science example: Plot monthly data for high and low temperatures in two locations, one coastal and one inland (e.g., San Francisco County vs. Sacramento). What patterns do you see? How can the influence of the ocean be seen in the observed patterns?**

In the early grades, the connections most frequently link individual science standards to individual math standards, as in the example above. As the grades progress, however, connections between the Common Core and the NGSS are often given for clusters of science and/or mathematics standards, rather than for individual standards. In other words, the Appendix might cite a series of science standards and relate those to more than one math standard. In these instances, the connections between science and math are far less clear.

Strengths

While there are some serious problems with the alignment between the Common Core math standards and the NGSS, several strengths are worth noting, particularly thanks to the addition of Appendix L (though this will only benefit the classroom if it is actually used in connection with NGSS). For starters, while few NGSS performance expectations include math—even when math might support science learning—Appendix L makes clear and explicit links between the NGSS and the CCSSM when such connections are made. Further, Appendix L includes science examples that demonstrate how grade-appropriate mathematics can be used to enhance and elucidate the science. Take, for example, the following, where a first-grade earth- and space-science standard (1-ESS1-2) is linked to two math standards:

> 1-ESS1-2. Make observations at different times of year to relate the amount of daylight to the time of year.
>
>> 1.OA.A.1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations to represent the problem. **Science example: There were 16 hours of daylight yesterday. On December 21, there were only 8 hours of daylight. How many more hours of daylight were there yesterday?**
>>
>> 1.MD.C.4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. **Science example: Based on the data we have collected so far on the bulletin board, which day has been the longest of the year so far? Which day has been the shortest of the year so far?**

Here, the math standard is clearly related to science learning, and educators are given clear guidance in the form of the "science example," showing the kinds of problems a teacher might ask students to deepen their understanding of science while making excellent use of grade-appropriate mathematics.

Similarly, in the following examples, the "science examples" demonstrate how students might apply math to understand the science required by the NGSS:

> 2-ESS1-1. Use information from several sources to provide evidence that Earth events can occur quickly or slowly.
>
>> 2.NBT.A. Understand place value. **Science example: As part of comprehending media to identify the varying timescales on which Earth events can occur, students understand that a period of**

thousands of years is much longer than a period of hundreds of years, which is in turn much longer than a period of tens of years.

3-LS3-2. Use evidence to support the explanation that traits can be influenced by the environment.

3.MD.B.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. **Science examples: (1) Make a line plot to show the height of each of a number of plants grown from a single parent. Observe that not all of the offspring are the same size. Compare the sizes of the offspring to the size of the parent. (2) Make a similar plot for plants grown with insufficient water.**

Finally, the following high school "connection" demonstrates how Appendix L makes clear connections between science and math learning:

HS-PS2-1. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

Interpreting Functions (F-IF) and Interpreting Categorical and Quantitative Data (S-ID). **Science examples: (1) Informally fit a quadratic function to the position-time data for a cart that rolls up an incline (slowing as it climbs, then reversing direction and speeding up as it descends). Use the algebraic expression for the fitted function to determine the magnitude of the cart's acceleration and initial speed. Over several trials, graph various quantities (such as acceleration vs. angle, or peak displacement vs. initial speed squared), and interpret the results. (2) Calculate and interpret the average speed of a moving object by using data from a distance-time graph.**

Weaknesses

While Appendix L adds significant value by more explicitly connecting the NGSS to the Common Core, it also suffers from several drawbacks. The most significant of those drawbacks fall into three categories:

- First, in several cases, NGSS expectations require math in order to fully understand the science content, but the math goes well beyond what students would have learned in classrooms aligned to the Common Core.
- Second, there are several "missed opportunities," where Appendix L could have made important links between grade-appropriate math and required science content but where no connections are made.
- Third, Appendix L too often makes "superficial connections," where grade-appropriate math is presented but does little to enhance science learning.

Alignment Glitches

In this section we look at NGSS performance expectations that require mathematics that is not yet available in the CCSSM at that grade level. Take, for example, the following:

> 4-PS3-1. Use evidence to construct an explanation relating the speed of an object to the energy of that object.

In order to do what this fourth-grade standard explicitly asks would require the use of quadratic functions. But quadratic functions are not studied until high school. One must assume that the NGSS authors did not intend the use of quadratic functions at this level—a point made clear by an "assessment boundary" that explicitly says that assessments should not be quantitative. But if math is prohibited, how exactly are students expected to "construct an explanation" in a situation that cannot be explained accurately without math?

Similarly, at fourth grade, students are asked to

> 4-PS4-1. Develop a model of waves to describe patterns in terms of amplitude and wavelength and that waves can cause objects to move.

Typically, the model for waves requires the use of trigonometric functions, something well beyond the reach of fourth graders. While the "assessment boundary" for this standard specifically excludes "quantitative models," the Common Core–connections column links to a supposedly relevant math standard. While Appendix L provides an interesting science example that uses the grade-appropriate math, neither the example nor the "connected" math standard give a model for waves or deal with amplitude or wavelength. Again, as written, the standard requires mathematics well above the grade level, but it apparently is supposed to mean something other than what it says.

At the middle school level, the topic of waves is revisited. This time, students are asked to

> MS-PS4-1. Use mathematical representations to describe a simple model for waves that includes how the amplitude of a wave is related to the energy in a wave.

As this standard is stated, it requires trigonometric and quadratic functions, both of which are high school mathematics. A clarification statement supports this requirement, stating that the "emphasis is on describing waves with both qualitative and quantitative thinking." The Common Core– connections column lists four related math standards, and again, Appendix L provides some very interesting science examples using grade-appropriate mathematics. Unfortunately, middle school mathematics cannot address a "mathematical representation" for waves or "how the amplitude of a wave is related to the energy." This standard requires the use of high school mathematics, but the NGSS evidently means something else, although just what remains unclear.

In fifth grade, students are asked to

> 5-ESS2-2. Describe and graph the amounts and percentages of water and fresh water in various reservoirs to provide evidence about the distribution of water on Earth.

Unfortunately, the Common Core doesn't introduce percentages until grade 6.

For middle school, we have the following:

> MS-PS2-2. Plan an investigation to provide evidence that the change in an object's motion depends on the sum of the forces on the object and the mass of the object.

As stated, this standard requires the equivalent of vectors, math that is not available in middle school. However, from the fine print, "Assessment is limited to forces and changes in motion in one-dimension…" In this case, the fine print contradicts the standard itself, perhaps in an effort to bring it into alignment with the CCSSM. The NGSS also provides three "connections" to Common Core math standards, and again, Appendix L includes several examples that apply to the modified one-dimensional standard. Unfortunately, while that is the only content possible using grade-appropriate math, it doesn't address the expectations in the standard itself. If the NGSS meant to ask for force in one dimension, that is how the standard should be stated.

A middle school earth- and space-science expectation asks students to

> MS-ESS2-6. Develop and use a model to describe how unequal heating and rotation of the Earth cause patterns of atmospheric and oceanic circulation that determine regional climates.

The mathematics of atmospheric dynamics is well beyond the reach of the average middle school student, high school student, or even college math major. Here, however, the assessment boundary puts no limits on the math that students should use. On the contrary, a clarification statement says,

> Emphasis is on how patterns vary by latitude, altitude, and geographic land distribution. Emphasis of atmospheric circulation is on the sunlight-driven latitudinal banding, the Coriolis effect, and resulting prevailing winds; emphasis of ocean circulation is on the transfer of heat by the global ocean convection cycle, which is constrained by the Coriolis effect and the outlines of continents.

After stating this grandiose standard in a way that appears to demand a very high level of mathematics, the NGSS contradicts itself and says, "Examples of models can be diagrams, maps and globes, or digital representations." This is equivalent to saying, "use mathematics but don't use mathematics." This makes no sense. The standard has to be taken at face value and, as such, is dramatically misaligned.

At the high school level, students are asked to

> HS-ESS1-4. Use mathematical or computational representations to predict the motion of orbiting objects in the solar system.

This draws upon rather serious college-level mathematics, and many engineering professionals make their living doing this. There is no possible appropriate alignment with CCSSM; the mathematics is at too high a level for K–12 education. The NGSS then undermines it own standard in the small print, saying that it "should not deal with more than two bodies, nor involve calculus." There is not much left of the "solar system" at this point, but the mathematics required for even this minimal interpretation of the standard (i.e., the mathematics of the ellipse) is STEM mathematics in the CCSSM (i.e., mathematics not required of all students and not to be assessed), and, as such, it is misaligned even at this lowest possible level.

Similarly, the following high school standards would require mathematics that exceeds what is included in the CCSSM, and the clarification statements and assessment boundaries do not constrain what should be expected of students.

> HS-ESS2-6. Develop a quantitative model to describe the cycling of carbon among the hydrosphere, atmosphere, geosphere, and biosphere.

No standard in the CCSSM even comes close to the mathematics needed for creating a "quantitative" model of the carbon cycle.

> HS-ESS3-3. Create a computational simulation to illustrate the relationships among management of natural resources, the sustainability of human populations, and biodiversity.

Again, there is nothing in the CCSSM that allows a student to “create a computational simulation” for anything so complex as biodiversity and the sustainability of human populations.

Missed Opportunities

Many standards in the NGSS would benefit from the use of mathematics but get no explicit guidance from either the "Common Core connections" column included in the NGSS or from Appendix L. Some of these standards mention math explicitly in the performance expectation, and the CCSSM has grade-appropriate standards that could be used to support the science. These are standards that use clear mathematical phrases like "analyze data," "make measurements," "use mathematical representations," "create a computational model," and "apply concepts of statistics and probability." Yet, despite a clear need (and opportunity) for mathematics to be specified to clarify these standards, Appendix L makes no connections. Take, for example, the following:

> K-PS2-2. Analyze data to determine if a design solution works as intended to change the speed or direction of an object with a push or a pull.
>
> 3-PS2-2. Make observations and/or measurements of an object's motion to provide evidence that a pattern can be used to predict future motion.
>
> 5-PS1-3. Make observations and measurements to identify materials based on their properties.
>
> MS-LS2-1. Analyze and interpret data to provide evidence for the effects of resource availability on organisms and populations of organisms in an ecosystem.

In other cases, there are broad "connections" between science and math mentioned in the connections column of the NGSS, but the examples provided in Appendix L do not address the content required in the standards. Take, for example, the following:

> HS-PS1-7. Use mathematical representations to support the claim that atoms, and therefore mass, are conserved during a chemical reaction.
>
> HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.
>
> HS-LS2 Ecosystems: Interactions, Energy, and Dynamics (8 standards, three of them start off with "Use mathematical representations.")
>
> HS-LS4-3. Apply concepts of statistics and probability to support explanations that organisms with an advantageous heritable trait tend to increase in proportion to organisms lacking this trait.

The standards themselves are not specific enough to give adequate guidance to educators. And the list of related math standards in the "connections" column don't go nearly far enough to guide curriculum and instruction. In order to make the link between math and science more explicit, Appendix L should have provided solid mathematical examples that meet the requirements of the standards. Otherwise, far too much is left open to interpretation.

Superficial Connections

Because the authors of Appendix L had the unenviable job of trying to retrofit mathematics into standards that did not incorporate much math in the first place, some attempts to create alignment and examples end up in grade-appropriate mathematics that does not contribute much to science learning. Take, for example, the following:

> 1-PS4-4. Use tools and materials to design and build a device that uses light or sound to solve the problem of communicating over a distance.
>
> 1.MD.A.1. Order three objects by length; compare the lengths of two objects indirectly by using a third object. **Science example: The class makes string phones. Maria's string is longer than Sue's...Sue's string is longer than Tia's...so without measuring directly we know that Maria's string is longer than Tia's.**

Here, the length of the string has little to do with solving a communications problem, the stated skill that students are supposed to master according to the NGSS.

> 1.MD.A.2. Express the length of an object as a whole number of length units, by layering multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. **Science example: Using a shoe as the length unit, the string for Sue's string phone is 11 units long.**

Again, this connection between science and math is artificial and does not deepen a student's understanding of science.

Similarly, in fourth grade, students are asked to

> 4-PS3-4. Apply scientific ideas to design, test, and refine a device that converts energy from one form to another.
>
>> 4.OA.A.3. Solve multistep word problems posed with whole numbers and having whole- number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Science example: The class has 144 rubber bands with which to make rubber-band cars. If each car uses six rubber bands, how many cars can be made? If there are 28**

> **students, at most how many rubber bands can each car have (if every car has the same number of rubber bands)?**

Computing the number of cars that can be made, although grade-appropriate math, makes no substantive contribution to creating “a device that converts energy from one form to another.”

In other examples, connections between math and science are made, but weakly, so the suggested science examples don’t really address the most important content or skills required in the NGSS. Consider the following:

HS-ESS3 Earth and Human Activity (6 standards)

> N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and origin in graphs and data displays.
>
> N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.
>
> N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
>
> **Science examples: (1) Quantify the impacts of human activities on natural systems. For example, if a certain activity creates pollution that in turn damages forests, then go beyond a qualitative statement by quantifying both the amount of pollution and the level of damage. (2) Carefully format data displays and graphs, attending to origin, scale, units, and other essential items.**

These science standards are about “explanation,” “evaluating,” “computational simulation,” and “computational representation.” Formatting data displays and graphs are, at best, minimal contributions to understanding the science.

Similar examples can be found throughout Appendix L.

Conclusion

Although Appendix L gives many worthwhile examples of grade-level mathematics applied to support and clarify the NGSS standards, it is, in essence, an after-the-fact corrective effort rather than the well-integrated mathematics that the NGSS should have produced. A fully satisfactory Appendix L would be a massive undertaking. Consequently, there are science standards that could have good mathematical connections but do not, either because the NGSS do not link math with them, the NGSS do link math with them but Appendix L doesn't clarify, or because the mathematics offered in Appendix L does not support the science in substantive ways. Appendix L could do nothing to deal with the NGSS that ask for mathematics that exceeds the corresponding grade level in the CCSSM. In these cases, we have serious misalignment rather than just a failure to find supporting alignment. Much of the NGSS document was not written with mathematics in mind. When mathematics was included, it generally lacked useful specificity. Consequently, Appendix L cannot and does not satisfactorily compensate.